TORSTEN HOEFLER

How fast will your application go? Static and dynamic techniques for application performance modeling

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Lausanne Switzerland | 08-10 June 2016

CLIMATE & WEATHER
SOLID EARTH
LIFE SCIENCE
CHEMISTRY & MATERIALS
PHYSICS
COMPUTER SCIENCE & MATHEMATICS
ENGINEERING
EMERGING DOMAINS

Performance modeling

- **What is this all about???**
  - A wide-spread practitioner’s view on performance modeling:

(replace “meeting” with performance optimization and “premeeting” with performance modeling)

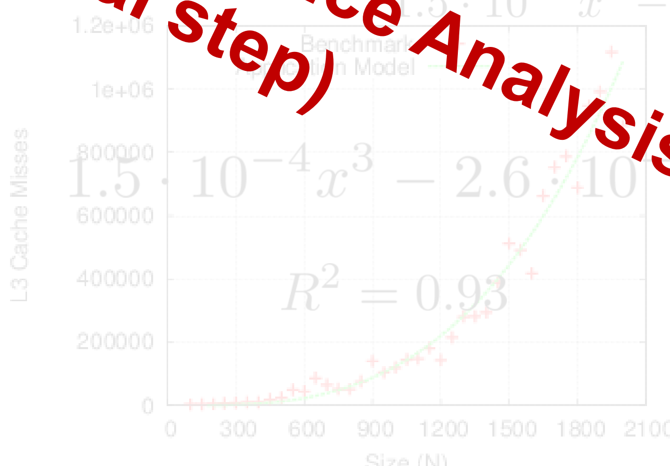
Performance Modeling = Performance Analysis v 2.0

(the next logical step)

Analytical application performance modeling

- **Scalability bug prediction**
  - Find latent scalability bugs early on (before machine deployment)
    SC13: A. Calotoiu, TH, M. Poke, F. Wolf: Using Automated Performance Modeling to Find Scalability Bugs in Complex Codes
- **Automated performance testing**
  - Performance modeling as part of a software engineering discipline in HPC
    ICS'15: S. Shudler, A. Calotoiu, TH, A. Strube, F. Wolf: Exascaling Your Library: Will Your Implementation Meet Your Expectations?
- **Hardware/Software co-design**
  - Decide how to architect systems
- **Making performance development intuitive**

vs.



$1.5 \cdot 10^{-4} x^3 - 2.6 \cdot 10^{-2} x^2$

$R^2 = 0.93$

Manual analytical performance modeling

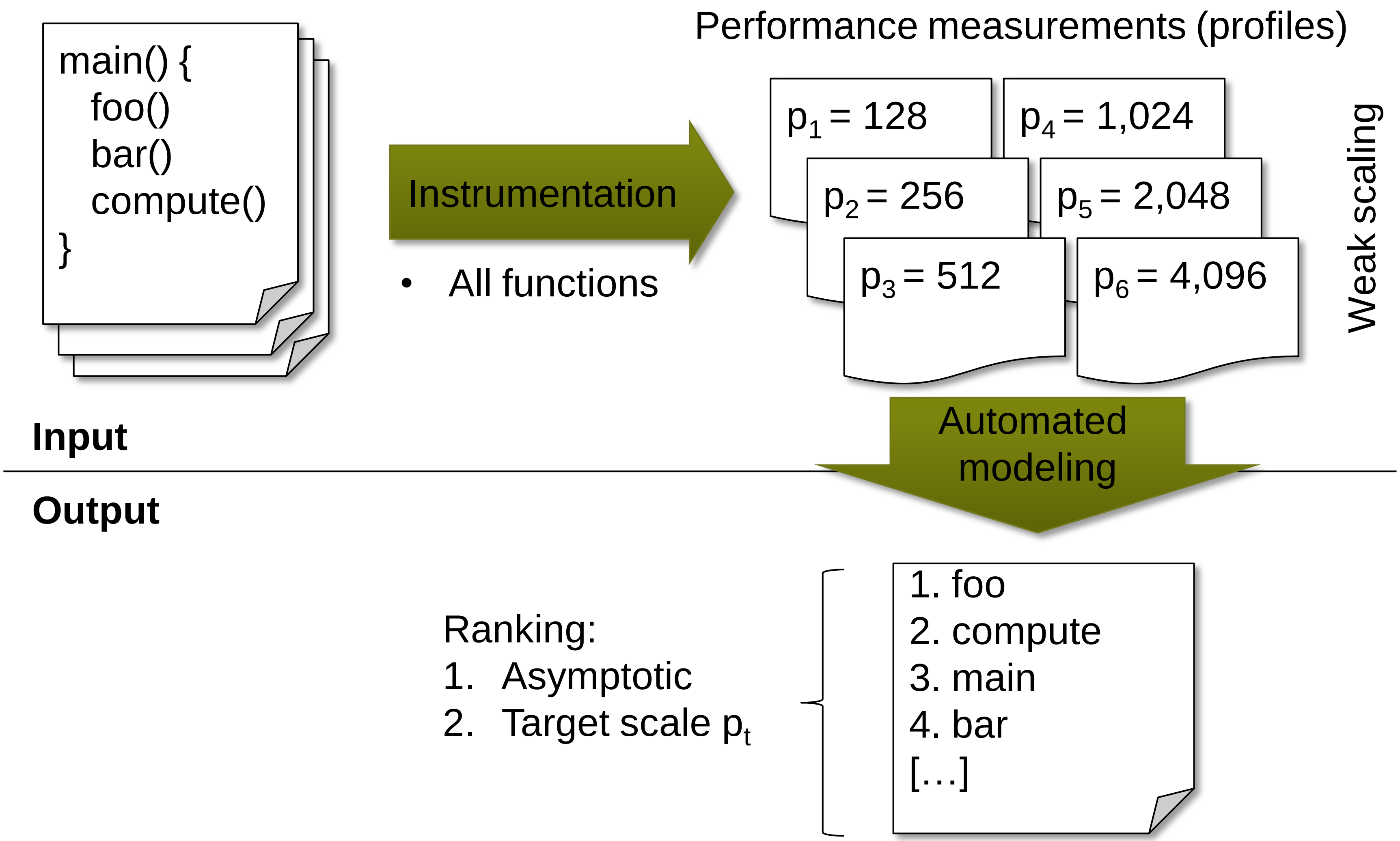
Identify kernels
- Parts of the program that dominate its performance at larger scales
- Identified via small-scale tests and intuition

Create models
- Laborious process
- Still confined to a small community of skilled experts

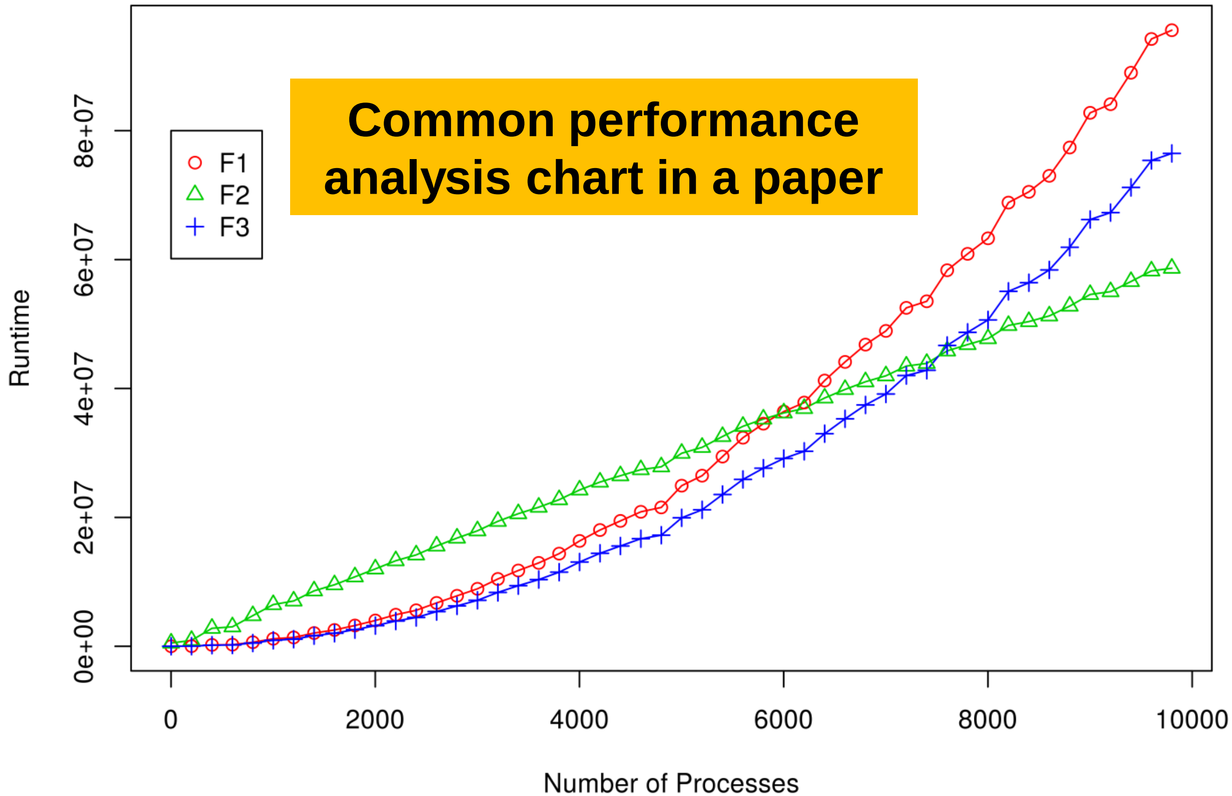
- **Disadvantages**
  - Time consuming
  - Error-prone, may overlook unscalable code

TH, W. Gropp, M. Snir, and W. Kramer: Performance Modeling for Systematic Performance Tuning, SC11

Our first step: scalability bug detector



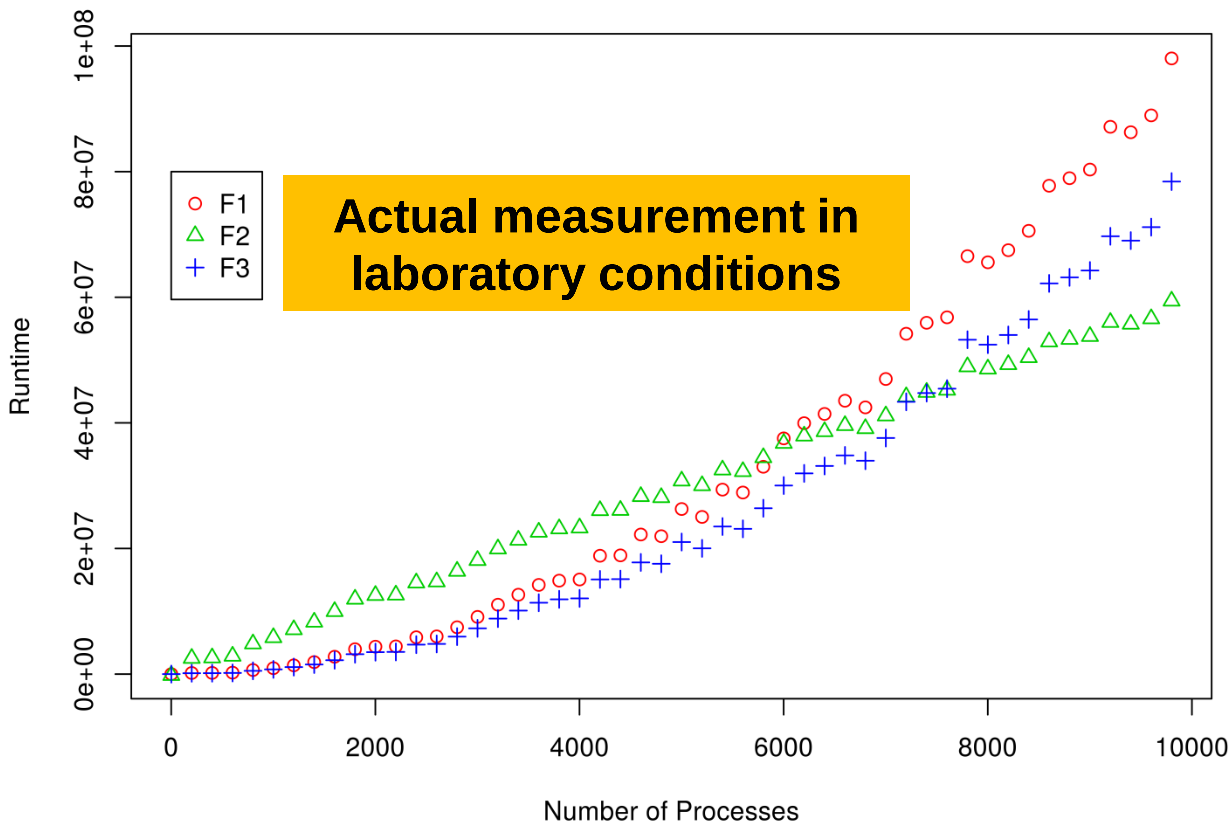
Primary focus on scaling trend



Our ranking

1. F_1
2. F_3
3. F_2

Primary focus on scaling trend



Our ranking

1. F_1
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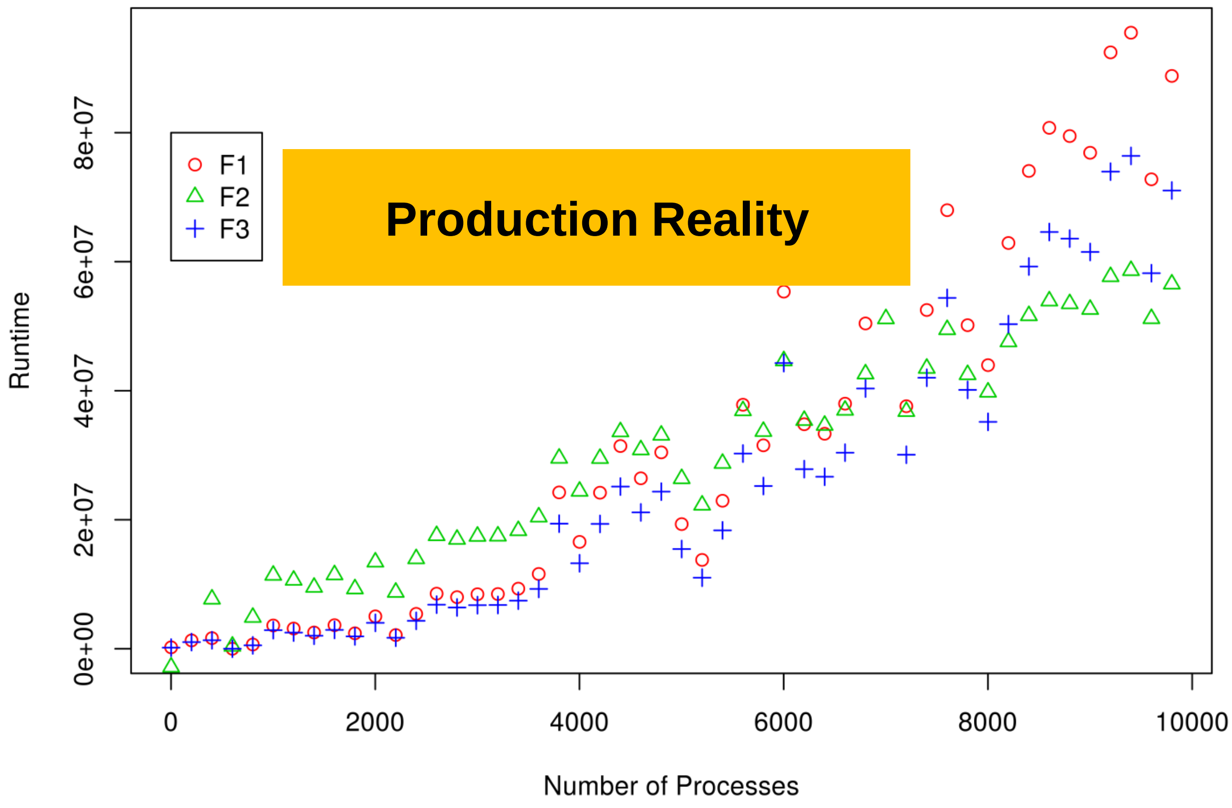
Primary focus on scaling trend



Our ranking

1. F_1
2. F_3
3. F_2

How to mechanize the expert? → Survey!

Computation

LU
$t(p) \sim c$

FFT
$t(p) \sim \log_2(p)$

Naïve N-body
$t(p) \sim p$

…

Samplesort
$t(p) \sim p^2 \log_2^2(p)$

Communication

LU
$t(p) \sim c$

FFT
$t(p) \sim \log_2(p)$

Naïve N-body
$t(p) \sim p$

…

Samplesort
$t(p) \sim p^2$

Survey result: performance model normal form

$$f(p) = \sum_{k=1}^{n} c_k \cdot p^{i_k} \cdot \log_2^{j_k}(p)$$

$n \in \mathbb{N}$

$i_k \in I$

$j_k \in J$

$I, J \subset \mathbb{Q}$

$n = 1$

$I = \{0, 1, 2\}$

$J = \{0, 1\}$

c_1

$c_1 \cdot p$

$c_1 \cdot p^2$

$c_1 \cdot \log(p)$

$c_1 \cdot p \cdot \log(p)$

$c_1 \cdot p^2 \cdot \log(p)$

Survey result: performance model normal form

$$f(p) = \sum_{k=1}^{n} c_k \cdot p^{i_k} \cdot \log_2^{j_k}(p)$$

$n \in \mathbb{N}$

$i_k \in I$

$n = 2$

$I = \{0, 1, 2\}$

$J = \{0, 1\}$

$c_1 + c_2 \cdot p$

$c_1 + c_2 \cdot p^2$

$c_1 + c_2 \cdot \log(p)$

$c_1 + c_2 \cdot p \cdot \log(p)$

$c_1 + c_2 \cdot p^2 \cdot \log(p)$

$c_1 \cdot \log(p) + c_2 \cdot p$

$c_1 \cdot \log(p) + c_2 \cdot p \cdot \log(p)$

$c_1 \cdot \log(p) + c_2 \cdot p^2$

$c_1 \cdot \log(p) + c_2 \cdot p^2 \cdot \log(p)$

$c_1 \cdot p + c_2 \cdot p \cdot \log(p)$

$c_1 \cdot p + c_2 \cdot p^2$

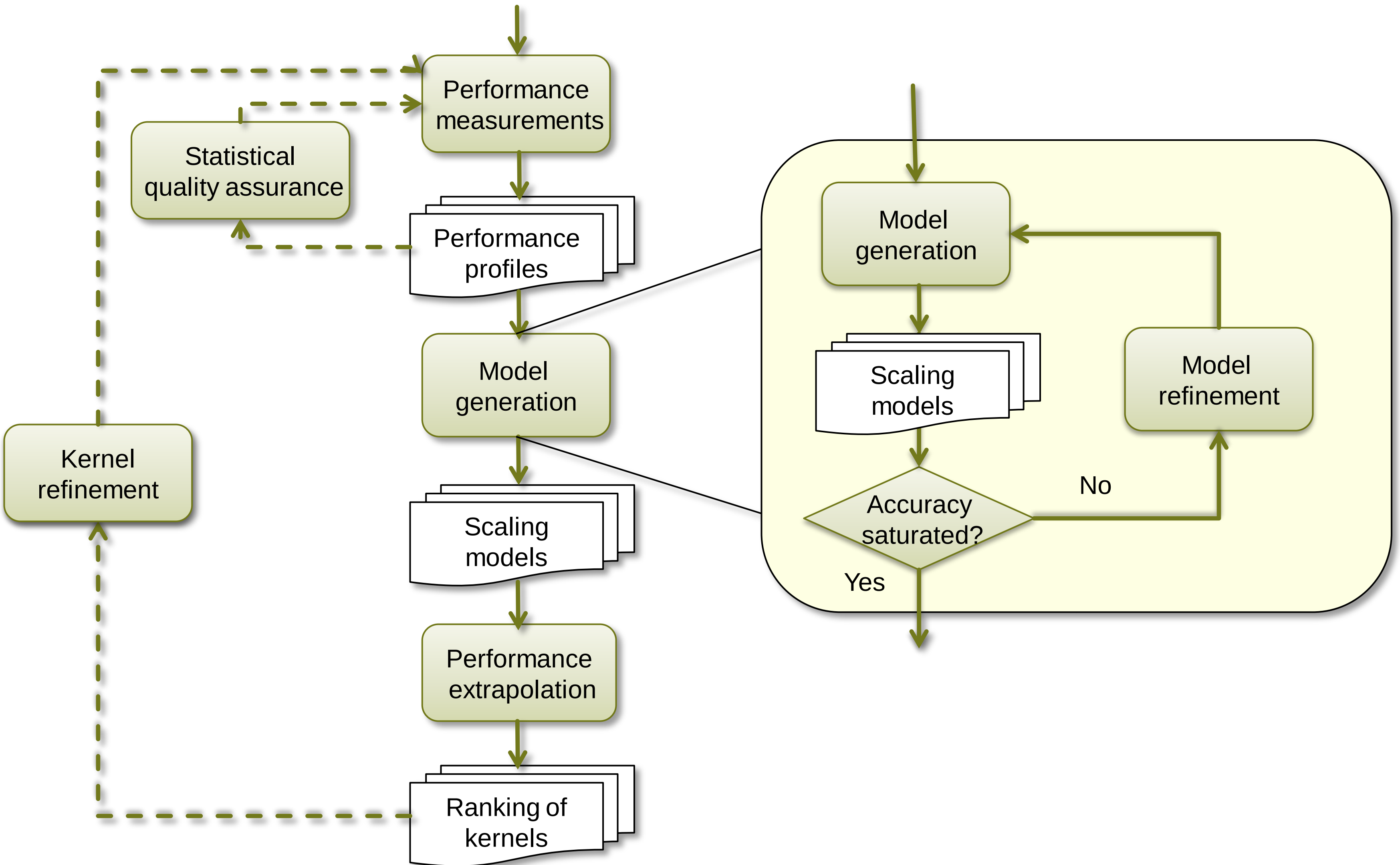
$c_1 \cdot p + c_2 \cdot p^2 \cdot \log(p)$

$c_1 \cdot p \cdot \log(p) + c_2 \cdot p^2$

$c_1 \cdot p \cdot \log(p) + c_2 \cdot p^2 \cdot \log(p)$

$c_1 \cdot p^2 + c_2 \cdot p^2 \cdot \log(p)$

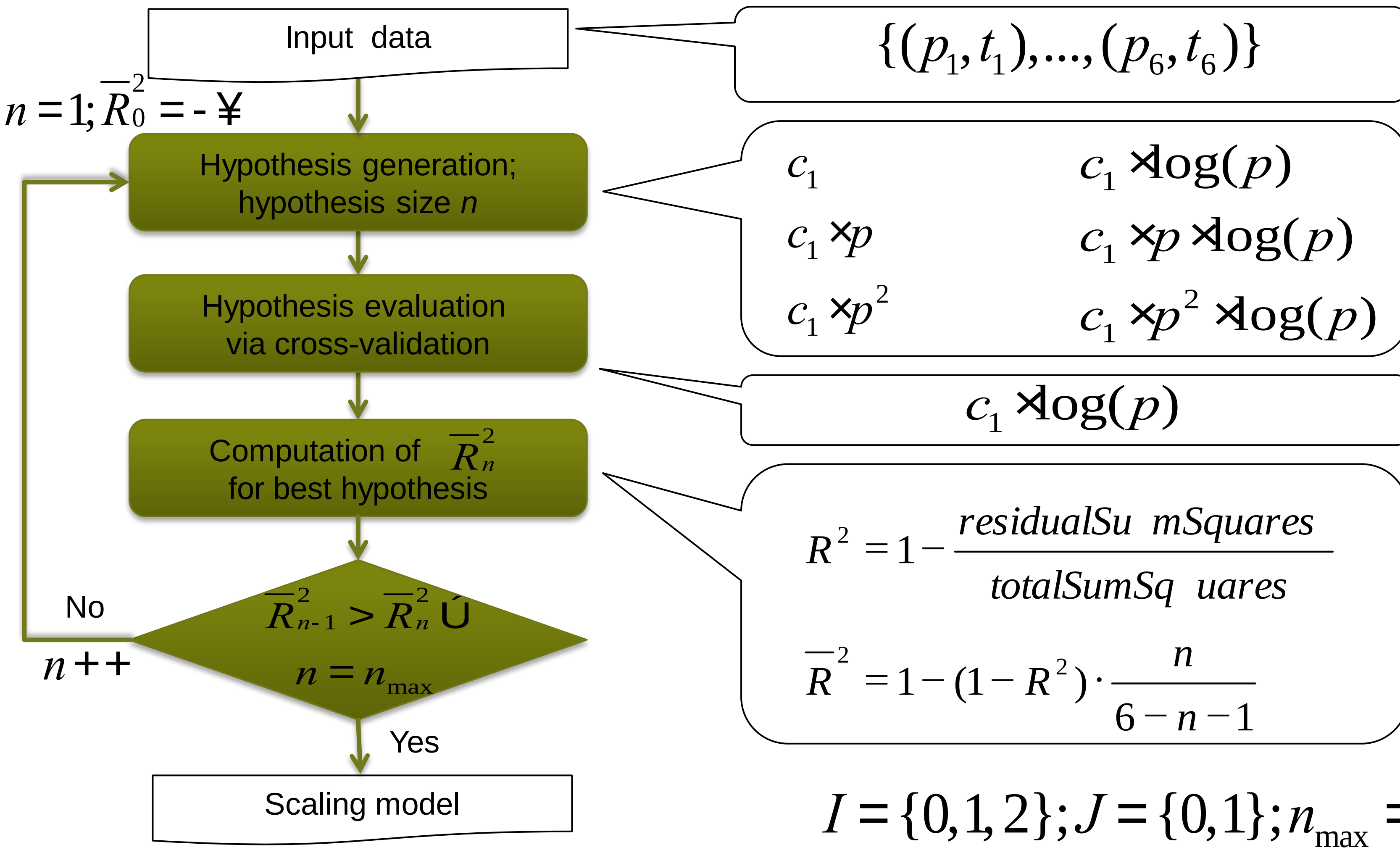
Our automated generation workflow



scalasca

A. Calotoiu, T. Hoefler, M. Poke, F. Wolf: Using Automated Performance Modeling to Find Scalability Bugs in Complex Codes, SC13

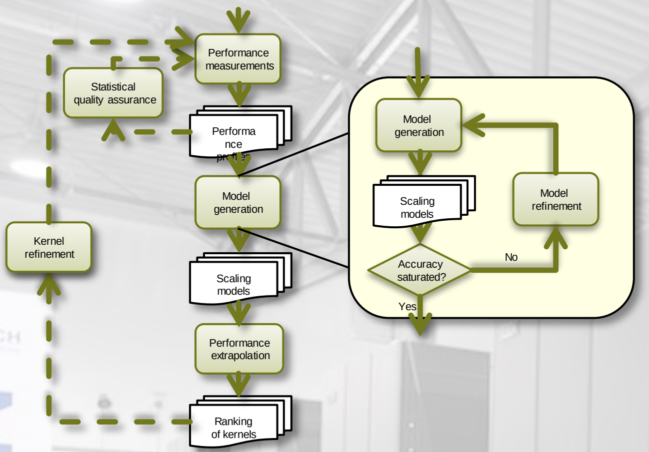
Model refinement



$$I = \{0, 1, 2\}; J = \{0, 1\}; n_{\max} = 2$$


JÜLICH
JUQUEEN
IBM
supercomputer Blue Gene Q

Evaluation overview

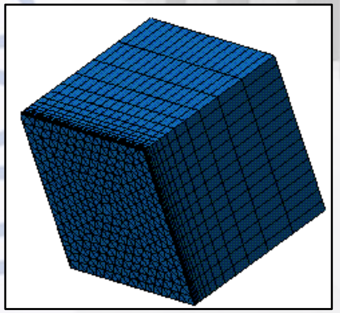


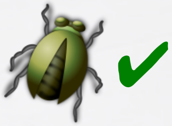
$$I = \{\tfrac{0}{2}, \tfrac{1}{2}, \tfrac{2}{2}, \tfrac{3}{2}, \tfrac{4}{2}, \tfrac{5}{2}, \tfrac{6}{2}\}$$

$$J = \{0, 1, 2\}$$

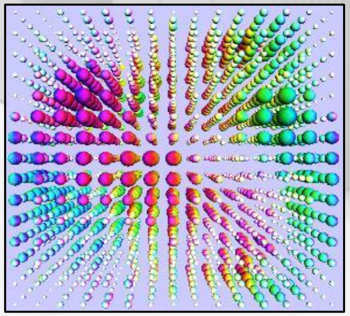
$$n = 5$$

Sweep3D



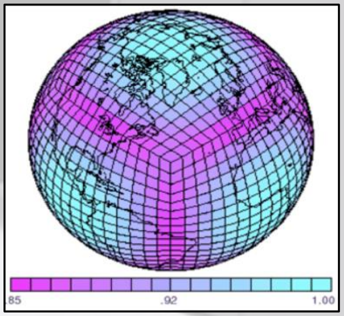


MILC

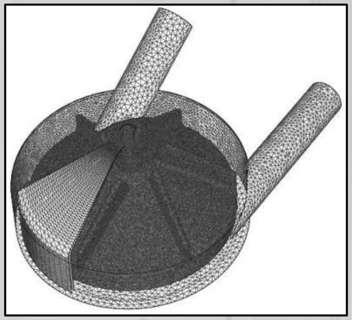


✖

HOMME

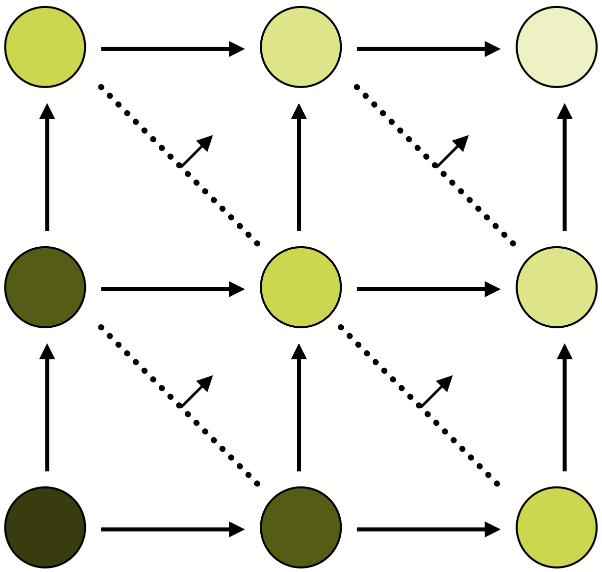


XNS



Sweep3D communication performance

- **Solves neutron transport problem**
- **3D domain mapped onto 2D process grid**
- **Parallelism achieved through pipelined wave-front process**

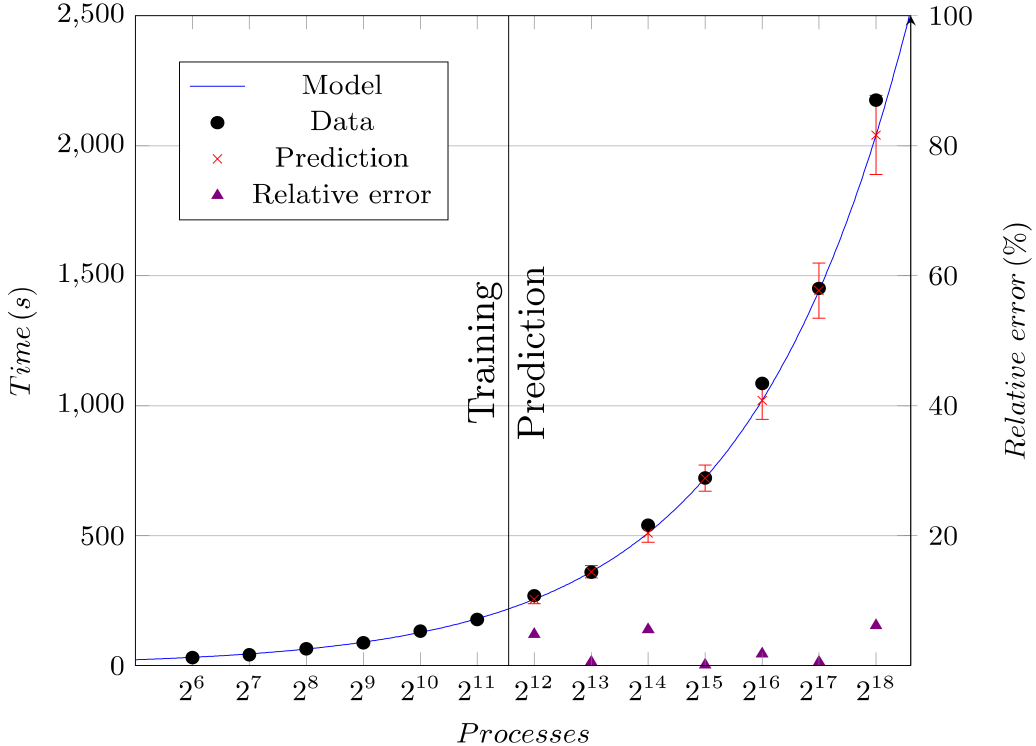


$$t^{comm} = c \cdot \sqrt{p}$$

- **LogGP model for communication developed by Hoisie et al.**
  - We assume $p=p_x*p_y$ → Equation (6) in [1]

[1] A. Hoisie, O. M. Lubeck, and H. J. Wasserman. Performance analysis of wavefront algorithms on very-large scale distributed systems. In Workshop on Wide Area Networks and High Performance Computing, pages 171–187. Springer-Verlag, 1999.

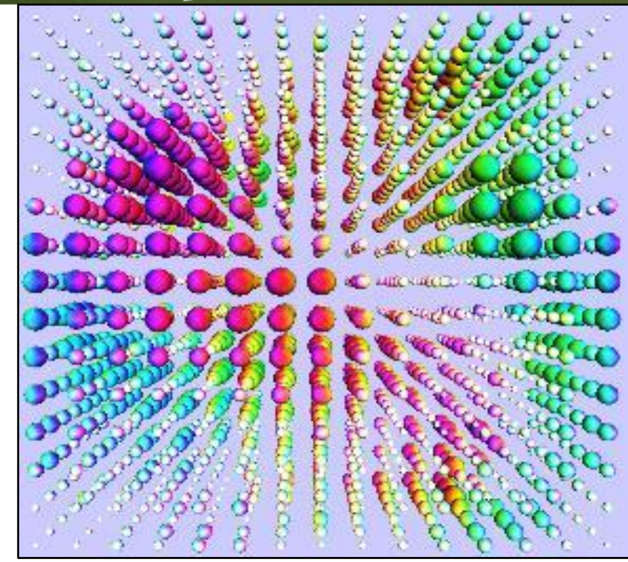
Sweep3D communication performance



Kernel [2 of 40]	Runtime[%] p_t=262k	Model [s] t = f(p)	Predictive error [%] p_t=262k
sweep → MPI_Recv	65.35	$4.03\sqrt{p}$	5.10
sweep	20.87	582.19	.01

#bytes = const.
#msg = const.

p_i £ 8k



MILC

- **MILC/su3_rmd – from MILC suite of QCD codes with performance model manually created**

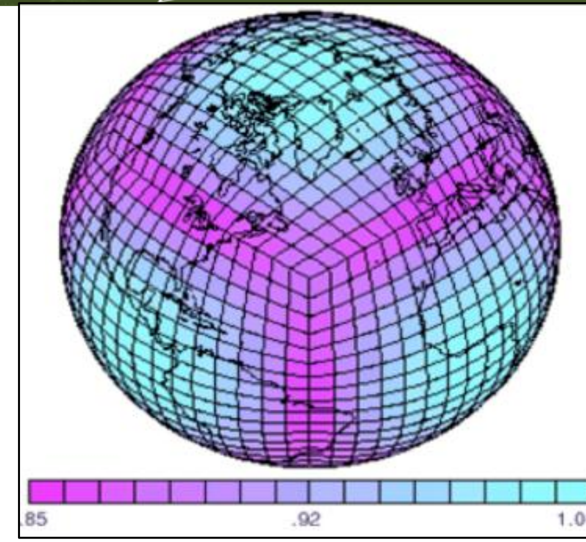
- **Time per process should remain constant except for a rather small logarithmic term caused by global convergence checks**

Kernel [3 of 479]	Model [s] t=f(p)	Predictive Error [%] p_t=64k
compute_gen_staple_field	2.40×10^{-2}	0.43
g_vecdoublesum → MPI_Allreduce	$6.30 \times 10^{-6} \times \log_2^2(p)$	0.01
mult_adj_su3_fieldlink_lathwec	3.80×10^{-3}	0.04

$$p_i \leq 16k$$

HOMME

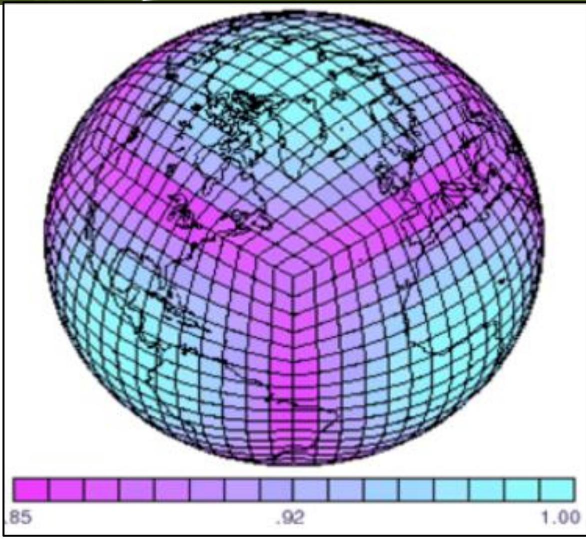
- **Core of the Community Atmospheric Model (CAM)**
- Spectral element dynamical core on a cubed sphere grid



Kernel [3 of 194]	Model [s] t = f(p)	Predictive error [%] $p_t = 130k$
box_rearrange → MPI_Reduce	$0.026 + 2.53 \times 10^{-6} p \times \sqrt{p} + 1.24 \times 10^{-12} p^3$	57.02
vlaplace_sphere_vk	49.53	99.32
compute_and_apply_rhs	48.68	1.65

$$p_i \le 15\text{k}$$

HOMME (2)

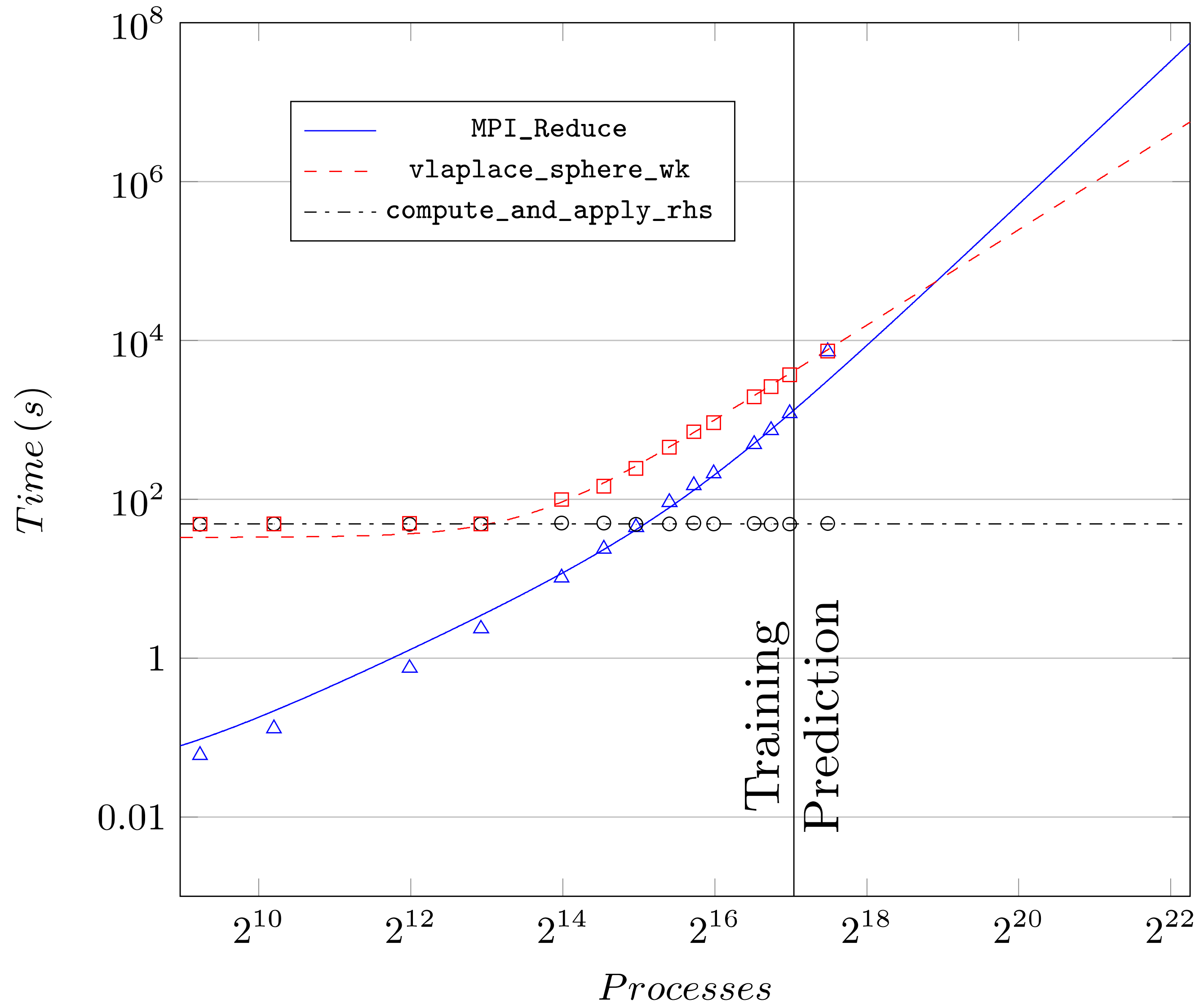


- **Core of the Community Atmospheric Model (CAM)**
- Spectral element dynamical core on a cubed sphere grid

Kernel [3 of 194]	Model [s] t = f(p)	Predictive error [%] p_t = 130k
box_rearrange → MPI_Reduce	$3.63\times10^{-6}p\times\sqrt{p}+7.21\times10^{-13}p^3$	30.34
vlaplace_sphere_vk	$24.44+2.26\times10^{-7}p^2$	4.28
compute_and_apply_rhs	49.09	0.83

p_i £ 43k

HOMME (3)


MPI_Reduce
vlaplace_sphere_wk
compute_and_apply_rhs
10^8
10^6
10^4
10^2
1
0.01
$Time\ (s)$
2^{10}
2^{12}
2^{14}
2^{16}
2^{18}
2^{20}
2^{22}
$Processes$
Training
Prediction

What about strong scaling?

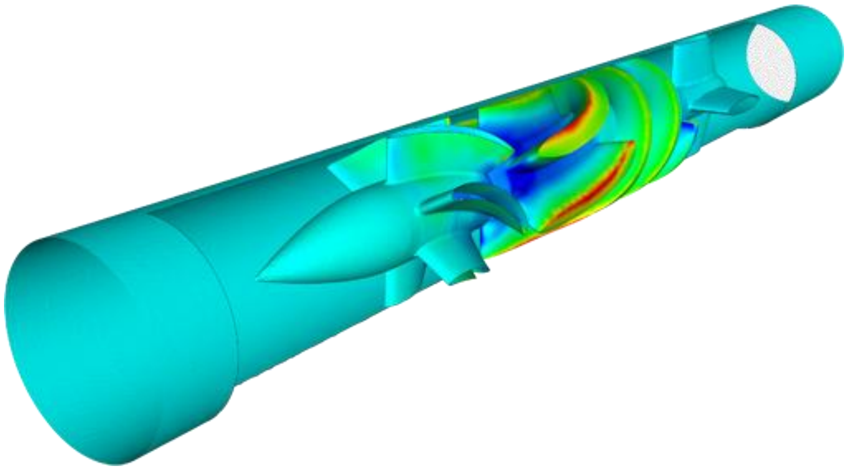
- **Wall-clock time not necessarily monotonically increasing – harder to capture model automatically**
  - Different invariants require different reductions across processes

	Weak scaling	Strong scaling
Invariant	Problem size per process	Overall problem size
Model target	Wall-clock time	Accumulated time
Reduction	Maximum / average	Sum

- **Superlinear speedup through cache effects**
  - Measure and model re-use distance?

XNS

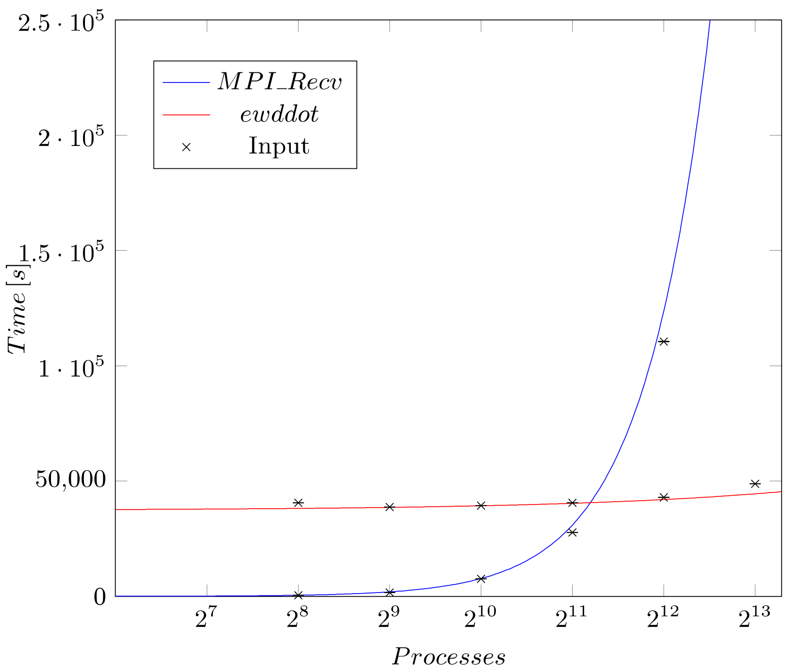
- **Finite element flow simulation program with numerous equations represented:**
  - Advection diffusion
  - Navier-Stokes
  - Shallow water



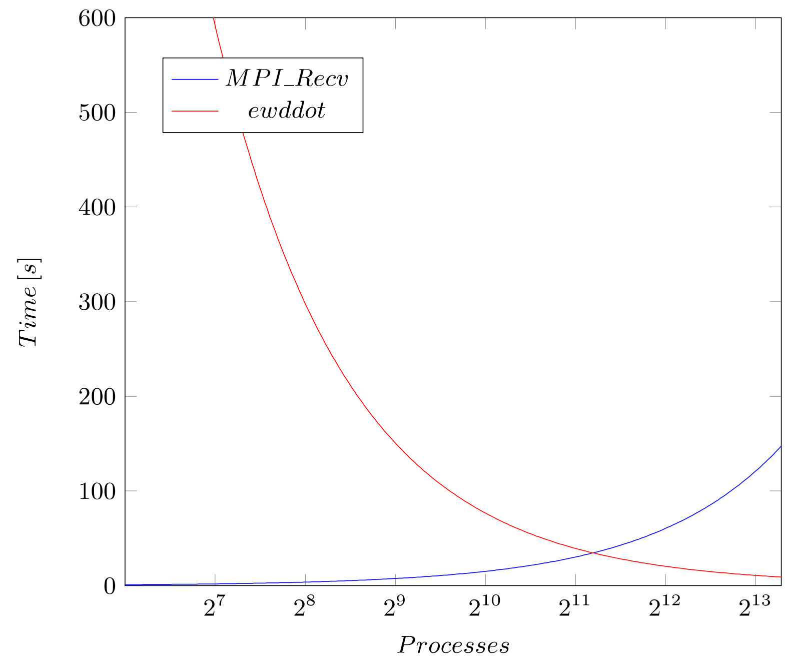
- **Strong scaling analysis**
  - P = {128; …; 4,096}
  - 5 measurements per p_i
  - Using accumulated time across processes as metric

XNS (2)

Accumulated time



Wallclock time

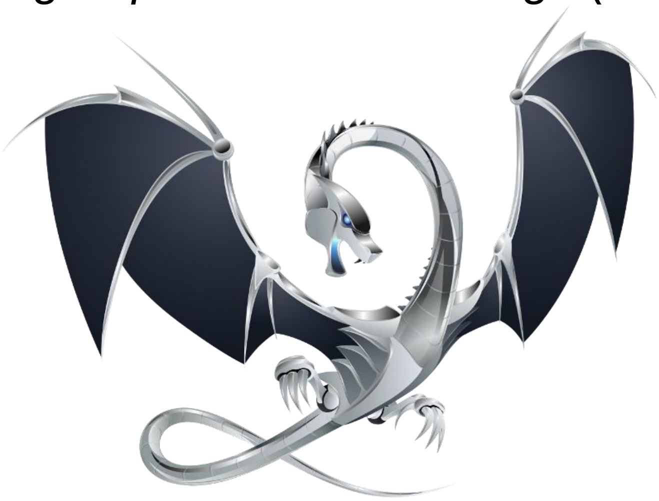


Kernel	Runtime[%] p=128	Runtime[%] p=4,096	Model [s] t = f(p)
ewdgennprm->MPI_Recv	0.46	51.46	$0.029 \times p^2$
ewddot	44.78	5.04	$p \times \log(p)$

#bytes = ~p
#msg = ~p

Is this all? No, it's just the beginning …

- **We face several problems:**
  - Multiparameter modeling – search space explosion
    Interesting instance of the curse of dimensionality
  - Modeling overheads
    Cross validation (leave-one-out) is slow and
    Our current profiling requires a lot of storage (>TBs)



Step back – what do we really care about?

- **Work**

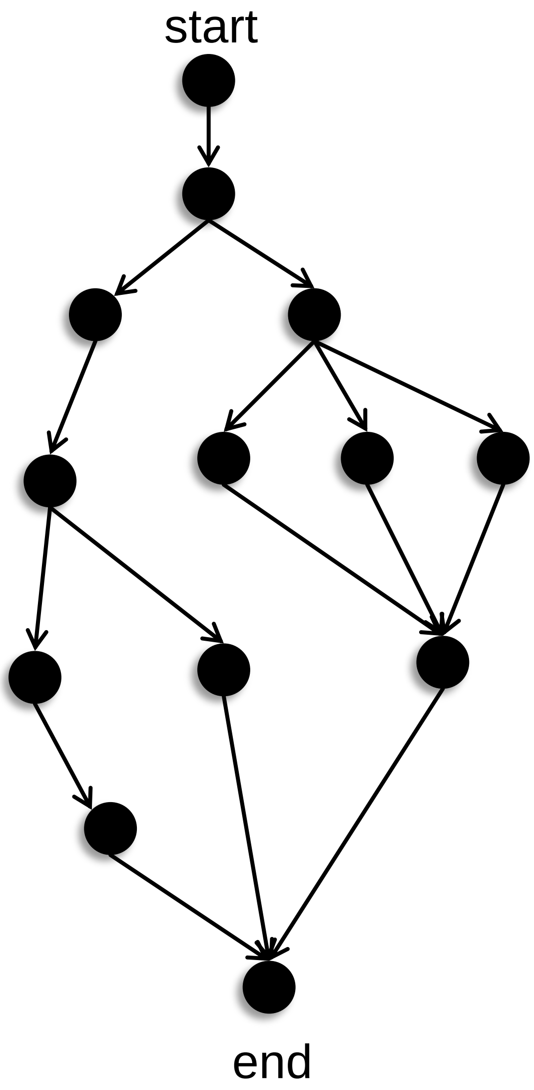
$$W = T_1$$

- **Depth**

$$D = T_\infty$$

- **Parallel efficiency**

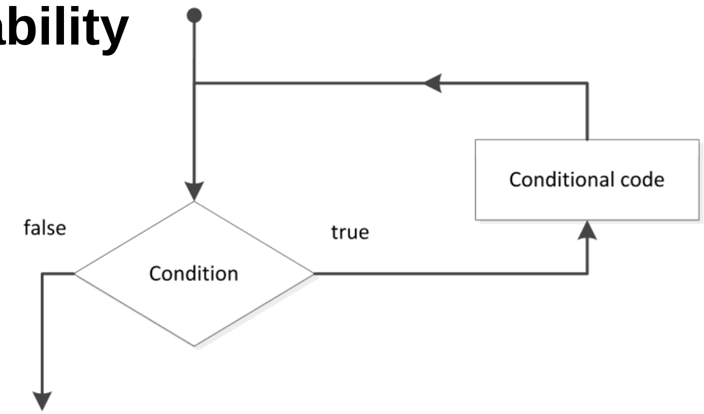
$$E_p = \frac{T_1}{pT_p}$$



Static analysis of explicitly parallel programs

- **Structures that determine program scalability**

LOOPS



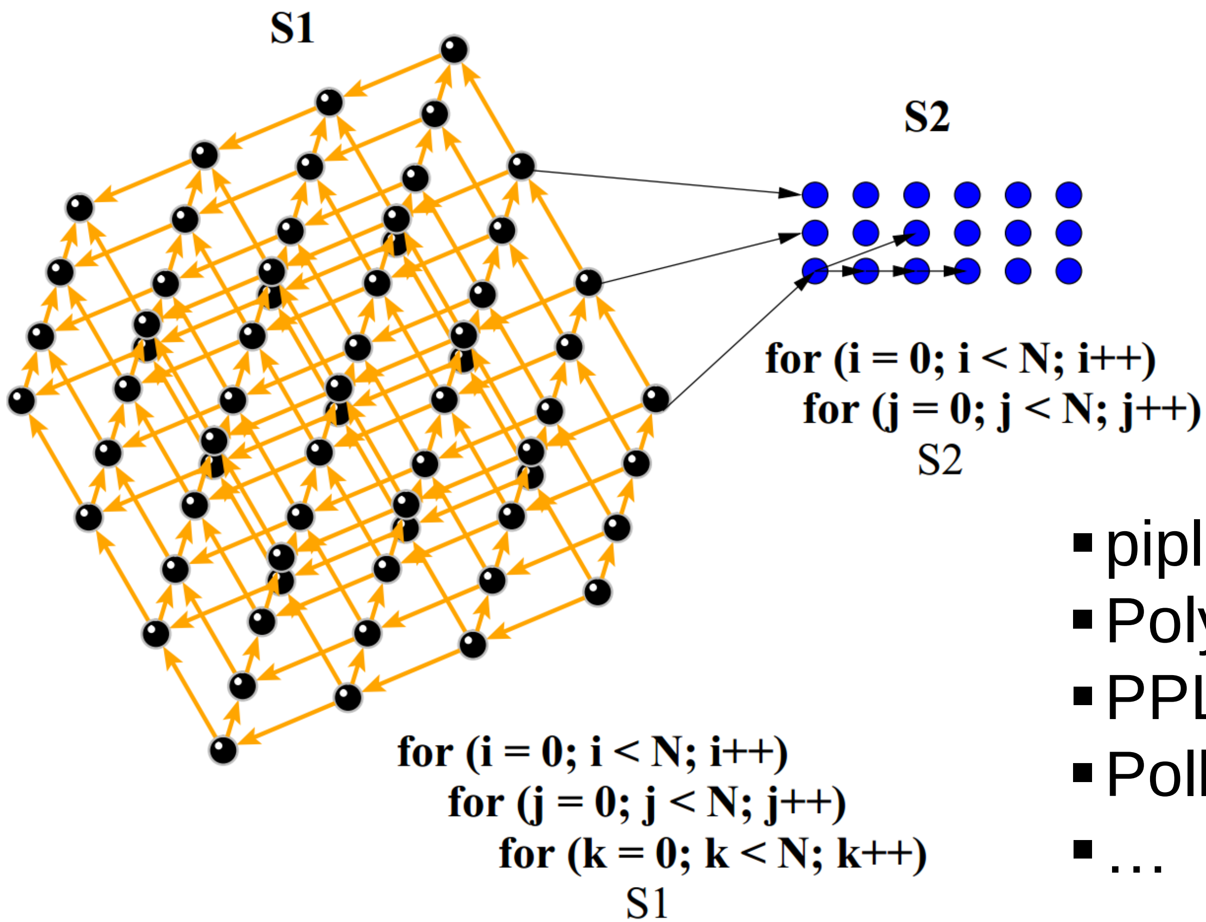
- **Assumption:**
  Other instructions do not influence it

- **Example:**

```
for (x=0; x < n/p; x++)
    for (y=1; y < n; y=2*y )
          veryComplicatedOperation(x,y);
```

Related work: counting loop iterations

- **Polyhedral model**



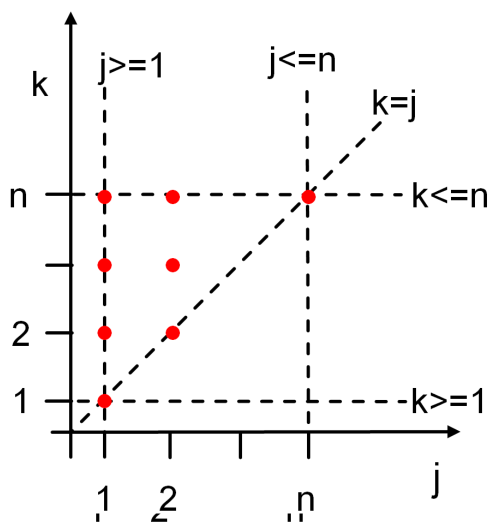
- piplib
- PolyLib
- PPL
- Polly
- …

R. Karp, R. Miller, and S. Winograd. The organization of computations for uniform recurrence equations. J. ACM, 14(3):563–590, July 1967.

Related work: counting loop iterations

- **Polyhedral model**

```
for (j = 1; j <= n; j = j*2)
        for (k = j; k <= n; k = k++)
                veryComplicatedOperation(j,k);
```



$$j \in [1, n]$$

$$k \in [j, n]$$

$$N = (n+1)\log_2 n - n + 2$$

$$N = \frac{n(n+1)}{2}$$

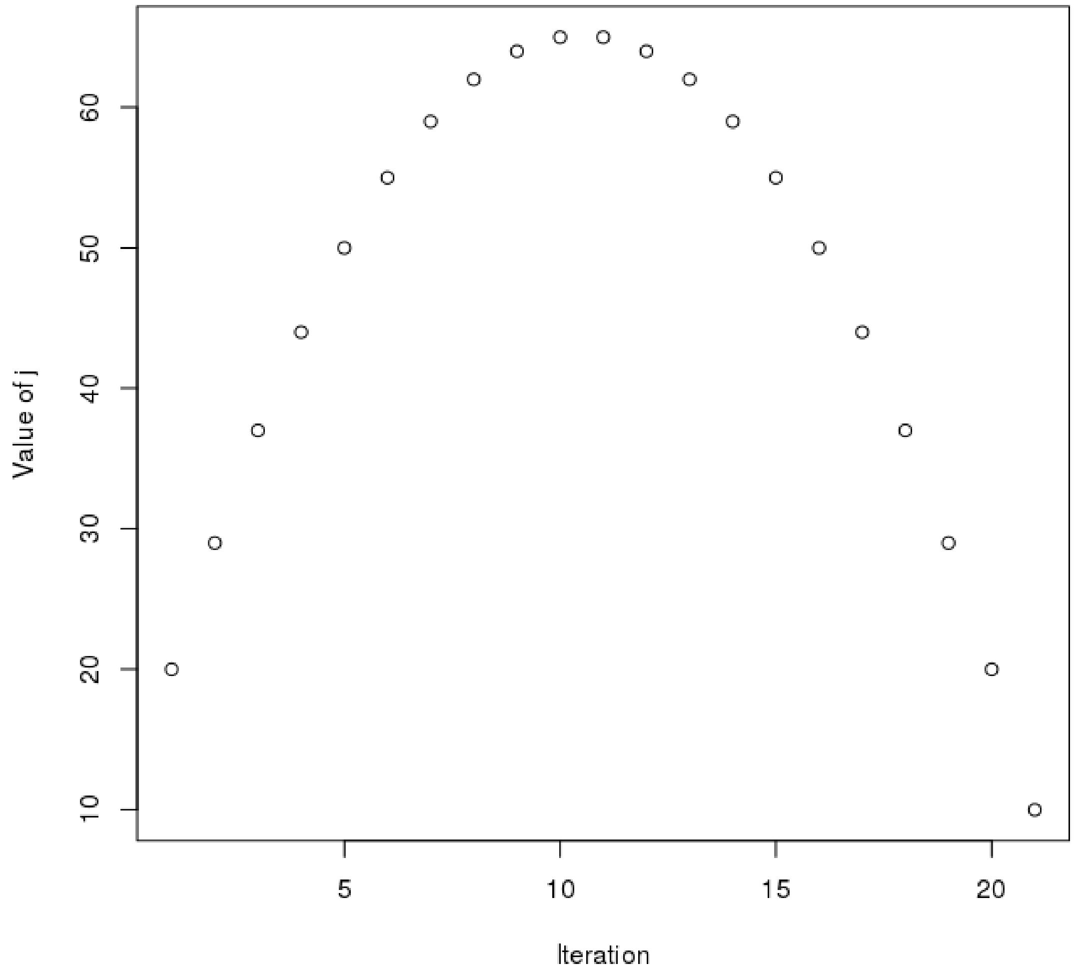
A.I. Barvinok. A polynomial time algorithm for counting integral points in polyhedra when the dimension is fixed, Math. Oper. Res., 1994

Related work: counting loop iterations

- **When the polyhedral model cannot handle it**

```
j=10;
k=10;
while (j>0){
    j=j+k;
    k--;
}
```

?



Counting arbitrary affine loop nests

- **Affine loops**

```
x=x_0;              // Initial assignment
while(c^T x < g)    // Loop guard
    x=Ax + b;       // Loop update
```

- **Perfectly nested affine loops**

```
while(c_1^T x < g_1) {
  x = A_1 x + b_1;
  while(c_2^T x < g_2) {
    ...
     x = A_{k-1} x + b_{k-1};
     while(c_k^T x < g_k) {
         x = A_k x + b_k;
         while(c_{k+1}^T x < g_{k+1}) {... }
         x = U_k x + v_k; }
     x = U_{k-1} x + v_{k-1};
    ...}
  x = U_1 x + v_1;}
```

$$A_k, U_k \in \mathbb{R}^{m\times m},\ b_k, v_k, c_k \in \mathbb{R}^m,\ g_k \in \mathbb{R} \text{ and } k = 1 \ldots r.$$

Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
        for (k=j; k < m; k = k + j )
                veryComplicatedOperation(j,k);
```

Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
    for (k=j; k < m; k = k + j )
        veryComplicatedOperation(j,k);
```

---

while($c_1^T x < g_1$) {
  $x = A_1 x + b_1;$
  while($c_2^T x < g_2$) {
    ...
    $x = A_{k-1} x + b_{k-1};$
    while($c_k^T x < g_k$) {
      $x = A_k x + b_k;$
      while($c_{k+1}^T x < g_{k+1}$) { ... }
      $x = U_k x + v_k;$ }
    $x = U_{k-1} x + v_{k-1};$
  ...}
  $x = U_1 x + v_1;$}

Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
    for (k=j; k < m; k = k + j )
        veryComplicatedOperation(j,k);
```

$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$\texttt{while}(c_1^T x < g_1)$ {
  $x = A_1 x + b_1;$
  $\texttt{while}(c_2^T x < g_2)$ {
    ...
    $x = A_{k-1} x + b_{k-1};$
    $\texttt{while}(c_k^T x < g_k)$ {
      $x = A_k x + b_k;$
      $\texttt{while}(c_{k+1}^T x < g_{k+1})$ {... }
      $x = U_k x + v_k;$ }
    $x = U_{k-1} x + v_{k-1};$
  ...}
  $x = U_1 x + v_1;$}

Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
    for (k=j; k < m; k = k + j )
        veryComplicatedOperation(j,k);
```

$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$$while(\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} j \\ k \end{pmatrix} < {n}/{p} + 1)\{$$

$$\}$$

$$
\begin{array}{l}
\texttt{while}(c_1^T x < g_1)\ \{ \\
\quad x = A_1 x + b_1; \\
\quad \texttt{while}(c_2^T x < g_2)\ \{ \\
\qquad \ldots \\
\qquad x = A_{k-1} x + b_{k-1}; \\
\qquad \texttt{while}(c_k^T x < g_k)\ \{ \\
\qquad\quad x = A_k x + b_k; \\
\qquad\quad \texttt{while}(c_{k+1}^T x < g_{k+1})\ \{\ldots\ \} \\
\qquad\quad x = U_k x + v_k;\ \} \\
\qquad x = U_{k-1} x + v_{k-1}; \\
\qquad \ldots\} \\
\quad x = U_1 x + v_1;\}
\end{array}
$$

Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
    for (k=j; k < m; k = k + j )
        veryComplicatedOperation(j,k);
```

while($c_1^T x < g_1$) {
  $x = A_1 x + b_1;$
  while($c_2^T x < g_2$) {
    ...
      $x = A_{k-1} x + b_{k-1};$
      while($c_k^T x < g_k$) {
        $x = A_k x + b_k;$
        while($c_{k+1}^T x < g_{k+1}$) {... }
        $x = U_k x + v_k;$ }
      $x = U_{k-1} x + v_{k-1};$
    ...}
  $x = U_1 x + v_1;$}

$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$$while\left(\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} j \\ k \end{pmatrix} < n/p + 1\right)\{$$

$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$while\left(\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} j \\ k \end{pmatrix} < m\right)\{$$

}

}

T. Hoefler, G. Kwasniewski: Automatic Complexity Analysis of Explicitly Parallel Programs, SPAA’14

Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
    for (k=j; k < m; k = k + j )
        veryComplicatedOperation(j,k);
```

$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$$\mathit{while}\left(\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} < n/p + 1\right)\{$$

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$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\}\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\}$$

---

while $(c_1^T x < g_1)$ {
  $x = A_1 x + b_1;$
  while $(c_2^T x < g_2)$ {
    ...
    $x = A_{k-1} x + b_{k-1};$
    while $(c_k^T x < g_k)$ {
      $x = A_k x + b_k;$
      while $(c_{k+1}^T x < g_{k+1})$ {... }
      $x = U_k x + v_k;$ }
    $x = U_{k-1} x + v_{k-1};$
  ...}
  $x = U_1 x + v_1;$}

T. Hoefler, G. Kwasniewski: Automatic Complexity Analysis of Explicitly Parallel Programs, SPAA'14

Counting arbitrary affine loop nests

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```
for (j=1; j < n/p + 1; j= j*2)
    for (k=j; k < m; k = k + j )
        veryComplicatedOperation(j,k);
```

$$
\begin{aligned}
&\texttt{while}(c_1^T x < g_1)\ \{\\
&\quad x = A_1 x + b_1;\\
&\quad \texttt{while}(c_2^T x < g_2)\ \{\\
&\qquad \ldots\\
&\qquad x = A_{k-1} x + b_{k-1};\\
&\qquad \texttt{while}(c_k^T x < g_k)\ \{\\
&\qquad\quad x = A_k x + b_k;\\
&\qquad\quad \texttt{while}(c_{k+1}^T x < g_{k+1})\ \{\ldots\ \}\\
&\qquad\quad x = U_k x + v_k;\ \}\\
&\qquad x = U_{k-1} x + v_{k-1};\\
&\qquad \ldots\}\\
&\quad x = U_1 x + v_1;\}
\end{aligned}
$$

$$
\begin{aligned}
&x = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix};\\
&while\left(\begin{pmatrix} 1 & 0 \end{pmatrix} x < {}^{n}\!/\!{}_{p} + 1\right)\{\\
&\quad x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};\\
&\quad while\left(\begin{pmatrix} 0 & 1 \end{pmatrix} x < m\right)\{\\
&\qquad x = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix}\\
&\quad \} x = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};\\
&\}
\end{aligned}
$$

where $x = \begin{pmatrix} j \\ k \end{pmatrix}$

Overview of the whole system

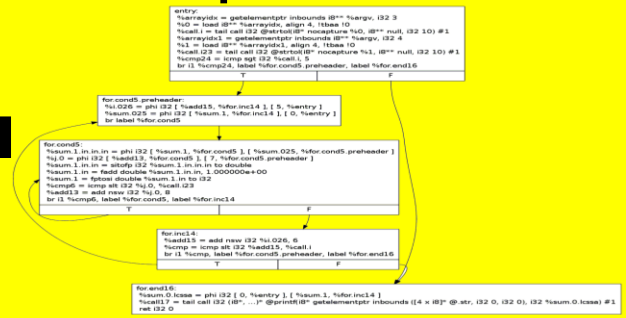
Parallel program

```
do i = 1, procCols
   call mpi_irecv( buff, 2, dp_type, reduce_exch_proc(i),
>                   i, mpi_comm_world, request, ierr )
   call mpi_send( buff2, 2, dp_type, reduce_exch_proc(i),
>                   i, mpi_comm_world, ierr )
   call mpi_wait( request, status, ierr )
enddo

do i = id *n/p, ( id +1)* n/p
   do j = 1, nSize
        call compute
```

LLVM

Loop extraction



Affine loop synthesis

$$
\begin{array}{l}
\texttt{while}(c_1^T x < g_1)\ \{\\
\quad x = A_1 x + b_1;\\
\quad \texttt{while}(c_2^T x < g_2)\ \{\\
\quad\quad \ldots\\
\quad\quad\quad x = A_{k-1} x + b_{k-1};\\
\quad\quad\quad \texttt{while}(c_k^T x < g_k)\ \{\\
\quad\quad\quad\quad x = A_k x + b_k;\\
\quad\quad\quad\quad \texttt{while}(c_{k+1}^T x < g_{k+1})\ \{\ldots\ \}\\
\quad\quad\quad\quad x = U_k x + v_k;\ \}\\
\quad\quad\quad x = U_{k-1} x + v_{k-1};\\
\quad\quad \ldots\}\\
\quad x = U_1 x + v_1;\}
\end{array}
$$

Closed form representation

$$x(i_1,\ldots,i_r) = A_{final}(i_1,\ldots,i_r) \cdot x_0 + b_{final}(i_1,\ldots,i_r)$$

with

$$i_r = 0 \ldots n_k(x_{0,k}),\ k = 1 \ldots r$$

Number of iterations

$$N = \sum_{i_1=0}^{n_1(x_{0,1})} \sum_{i_2=0}^{n_2(x_{0,2})} \cdots \sum_{i_{r-1}=0}^{n_{r-1}(x_{0,r-1})} n_r(x_{0,r})$$

Program analysis

$$W = N\big|_{p=1}$$

$$D = N\big|_{p\to\infty}$$

Algorithm in details

Closed form representation of a loop

- Single affine statement

$$x = Lx + p$$

- Counting function

$$n(x_0)$$

$$x(i, x_0) = L(i) \cdot x_0 + p(i)$$

$$n(x_0, c, g) = \arg\min_i \left(c^T \cdot x(i, x_0) \geq g\right)$$

- **Example**

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$while\ (1^T x < m)\{$$

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\}$$

$$x = x_0;$$

$$while\ (c^T x < g)$$

$$x = Ax + b;$$

$$x(i, x_0) = A^i x_0 + \sum_{j=0}^{i-1} A^j \cdot b$$

$$x(i, x_0) = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^i x_0 + \sum_{j=0}^{i-1} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^j \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} x_0 + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$n(x_0) = \left\lceil \frac{m - k_0}{j_0} \right\rceil$$

Algorithm in details

Folding the loops

$$x = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$$while\ \left(\begin{pmatrix} 1 & 0 \end{pmatrix} x < n/p\right)\{$$

$$\quad x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\quad while\ \left(\begin{pmatrix} 0 & 1 \end{pmatrix} x < m\right)\{$$

$$\qquad x = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

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$$\}$$

Algorithm in details

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$$\}$$

→

$$x = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$$\textit{while } (\begin{pmatrix} 1 & 0 \end{pmatrix} x < n/p)\{$$

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$x = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$x = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\}$$

Algorithm in details

Folding the loops

$$x = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

while $\left(\begin{pmatrix} 1 & 0 \end{pmatrix} x < n/p\right)\{$

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

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$$x = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

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$\}$

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$\}$

$$x = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

while $\left(\begin{pmatrix} 1 & 0 \end{pmatrix} x < n/p\right)\{$

$$x = \begin{pmatrix} 2 & 0 \\ i+1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$\}$

Algorithm in details

Starting conditions

$x_{0,1} \longrightarrow$ $x = x_0;$

$while\ (c_1^T x < g_1)\{$

$x_{0,2} \longrightarrow$ $x = A_1 x + b_1;$

$while\ (c_2^T x < g_2)\{$

$x_{0,3} \longrightarrow$ $x = A_2 x + b_2;$

$while\ (c_3^T x < g_3)\{$

$x = A_3 x + b_3;$

$\}x = U_2 x + v_2;$

$\}x = U_1 x + v_1;$

$\}$

Algorithm in details

Counting the number of iterations

We have:

Algorithm in details

Counting the number of iterations

We have:

- The closed form for each loop:
  - *Single affine statement*
  - *Counting function*
- Starting condition for each loop

Algorithm in details

Counting the number of iterations

We have:

- The closed form for each loop:
  - *Single affine statement*
  - *Counting function*
- Starting condition for each loop

Number of iterations:

$$N = \sum_{i_1=0}^{n_1(x_{0,1})} \sum_{i_2=0}^{n_2(x_{0,2})} \ldots \sum_{i_{r-1}=0}^{n_{r-1}(x_{0,r-1})} n_r(x_{0,r}).$$

Algorithm in details

Counting the number of iterations

- The equation computes the precise number of iterations

$$N = \sum_{i_1=0}^{n_1(x_{0,1})} \sum_{i_2=0}^{n_2(x_{0,2})} \dots \sum_{i_{r-1}=0}^{n_{r-1}(x_{0,r-1})} n_r(x_{0,r}).$$

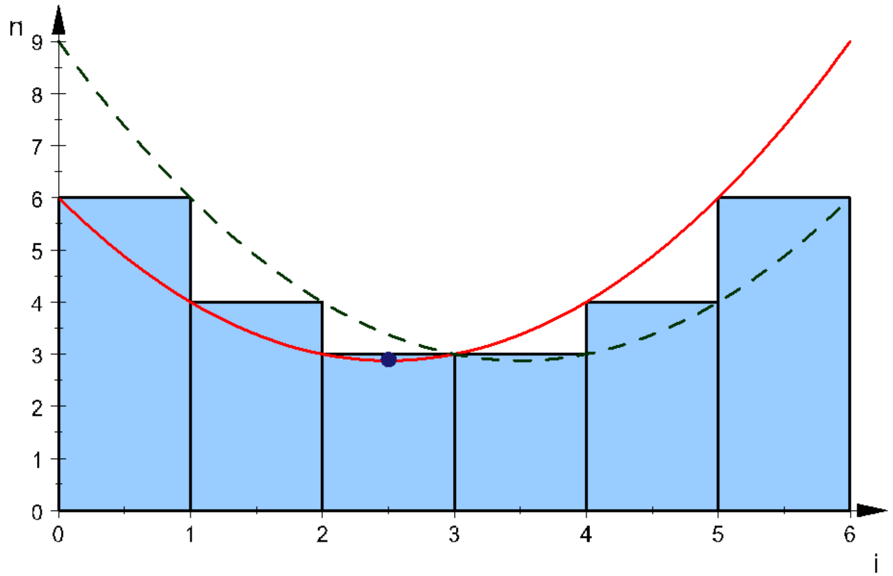
Algorithm in details

Counting the number of iterations

- The equation gives precise number of iterations

$$N = \sum_{i_1=0}^{n_1(x_{0,1})} \sum_{i_2=0}^{n_2(x_{0,2})} \dots \sum_{i_{r-1}=0}^{n_{r-1}(x_{0,r-1})} n_r(x_{0,r}).$$

- But simplification may fail → Sum approximation
  - *Approximate sums by integrals*
    → lower and upper bounds



Solving more general problems

Solving more general problems

- **Multipath loops**

Solving more general problems

- Multipath loops
- Conditional statements

Solving more general problems

- Multipath loops
- Conditional statements
- Non-affine loops

```
do j=1,lastrow-firstrow+1
   sum = 0.d0
   do k=rowstr(j),rowstr(j+1)-1
      sum = sum + a(k)*p(colidx(k))
   enddo
   w(j) = sum
enddo
```

$\texttt{lastrow-firstrow+1} = \texttt{row\_size} = \frac{\texttt{na}}{\texttt{nprows}}$

$\texttt{rowstr(j+1)-1-rowstr(j)} = u$

$$N = \frac{\texttt{na} \cdot u}{\texttt{nprows}}$$

Case studies

- **NAS Parallel Benchmarks: EP**

$$N(m,p) = \left\lceil \frac{2^{m-16} \cdot (u + 2^{16})}{p} \right\rceil$$

```
u :    do i=1,100
         ik =kk/2
         if (ik .eq. 0) goto 130
         kk=ik
       continue
```

Case studies

- **NAS Parallel Benchmarks: EP**

$$N(m,p) = \left\lceil \frac{2^{m-16} \cdot (u + 2^{16})}{p} \right\rceil$$

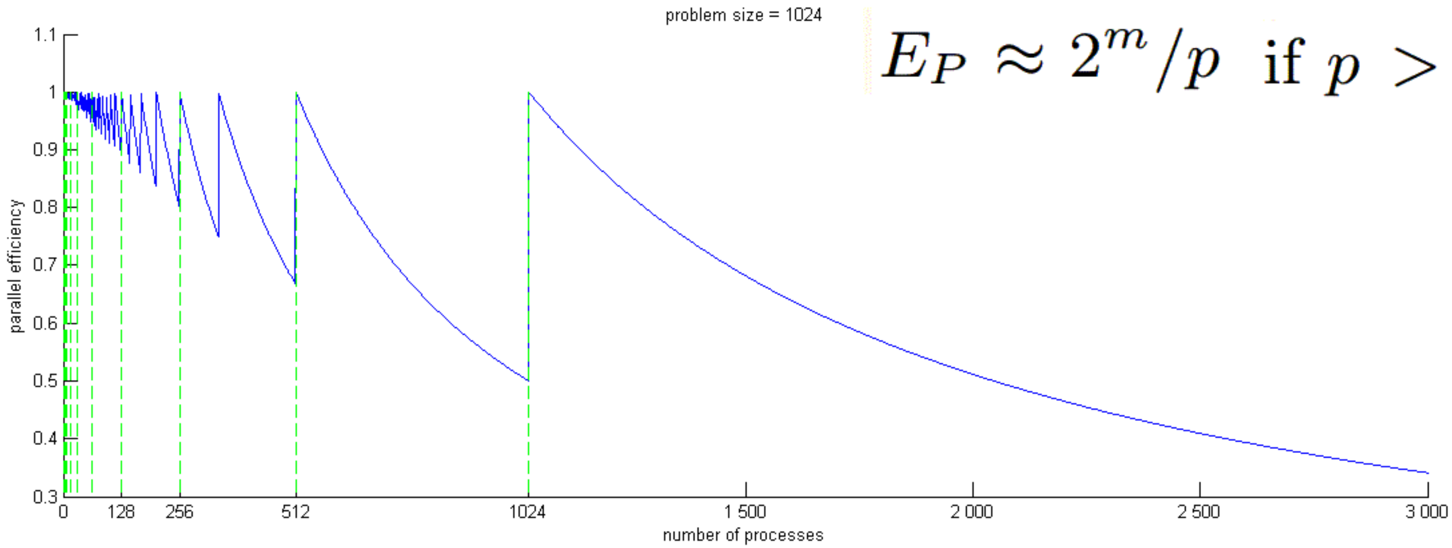
```
u :   do i=1,100
        ik =kk/2
        if (ik .eq. 0) goto 130
        kk=ik
      continue
```

$$W = T_1 \approx 2^m$$
$$D = T_\infty \approx 1$$

$$E_P = \frac{2^m}{p \left\lceil \frac{2^m}{p} \right\rceil}$$

$$E_P \approx 1 \text{ if } p \le 2^m$$

$$E_P \approx 2^m/p \text{ if } p > 2^m$$



Case studies

CG – conjugate gradient

$$W \approx k_1 \left\lceil \frac{m}{p} \right\rceil + k_2 \sqrt{\left\lceil \frac{m}{p} \right\rceil} + k_3 \log_2 \left(\sqrt{p} \right)$$

$$D = T_\infty = \infty$$

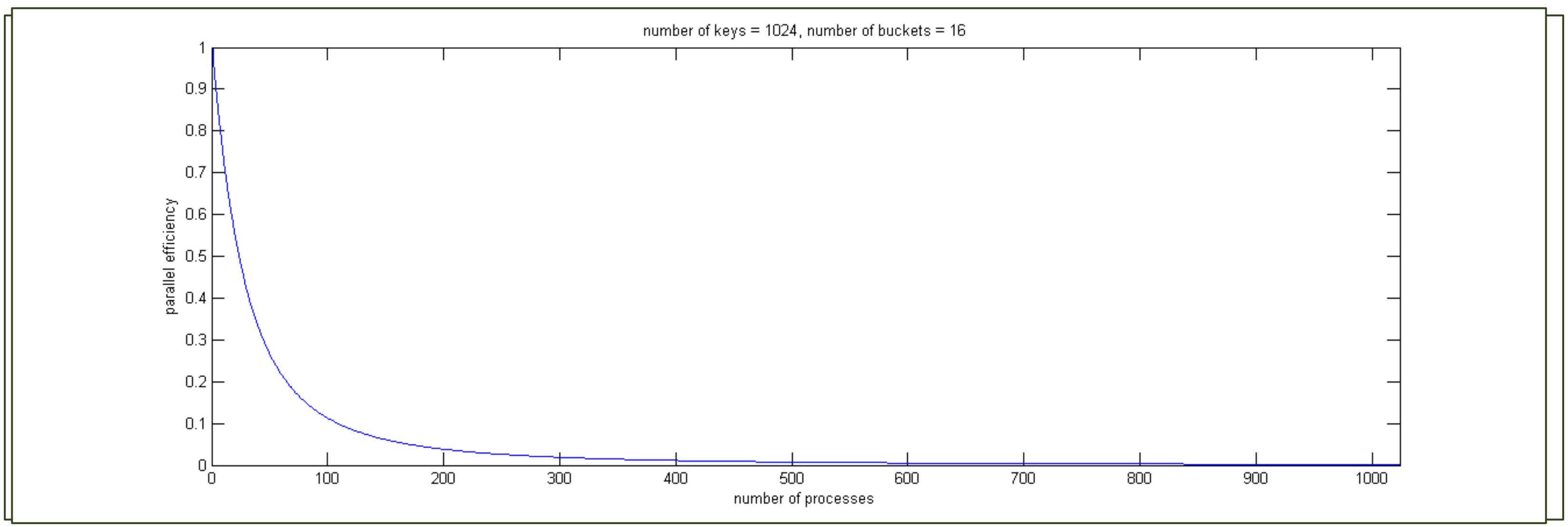
$$E_p = \frac{k_4}{p\left(k_1 \left\lceil \frac{m}{p} \right\rceil + k_2 \sqrt{\left\lceil \frac{m}{p} \right\rceil} + k_3 \log_2 \left(\sqrt{p} \right) \right)}$$

IS – integer sort

$$W \approx n\left(3(s+t) + 2\left\lceil \frac{m}{p} \right\rceil + p + u_1 + u_2 \right)$$

$$D = T_\infty = \infty$$

$$E_p = \frac{k_1}{p\left(p + k_2 \left\lceil \frac{m}{p} \right\rceil + k_3 \right)}$$



What problems are remaining?

- **Well, what about non-affine loops?**
  - More general abstract interpretation (next step)
  - Not solvable → will always have undefined terms

$$N = \frac{\mathtt{na} \cdot u}{\mathtt{nprows}}$$

- **Back to PMNF?**
  - Generalize to multiple input parameters

    a) Bigger search-space ☹

    b) Bigger trace files ☹

- **Ad-hoc (partial) solution: online machine learning – PEMOGEN**
  - Replace cross-validation with LASSO (regression with L_1 regularizer)

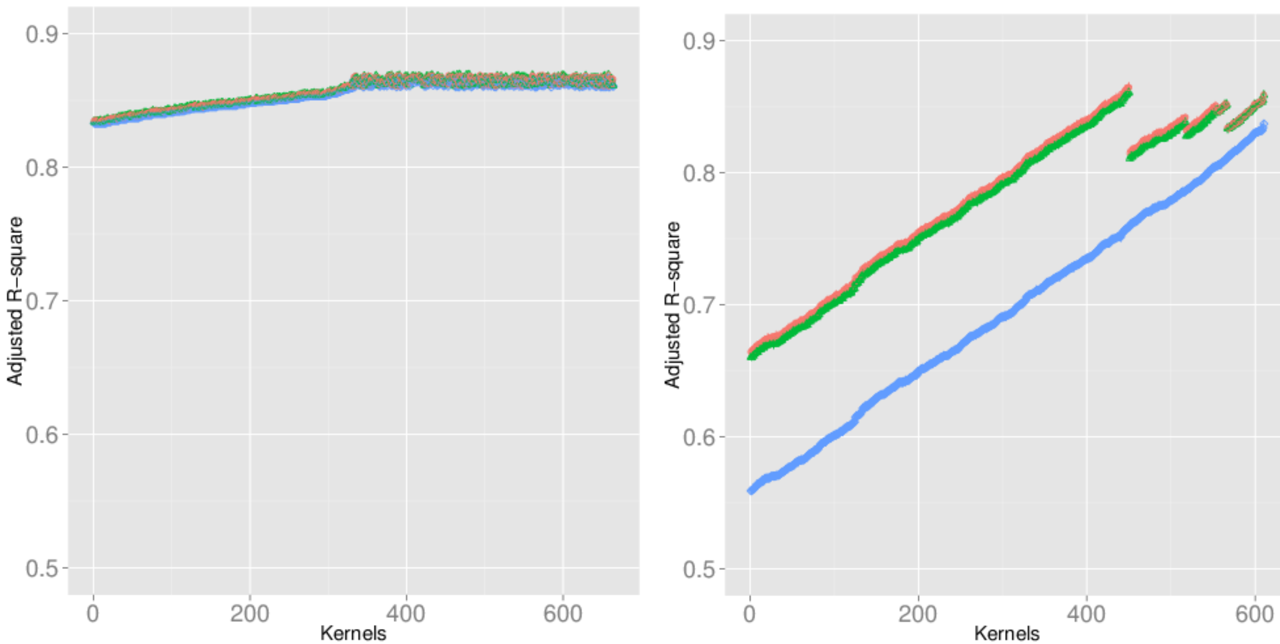
    Much cheaper!
  - Replace LASSO with online LASSO [1]

    No traces!

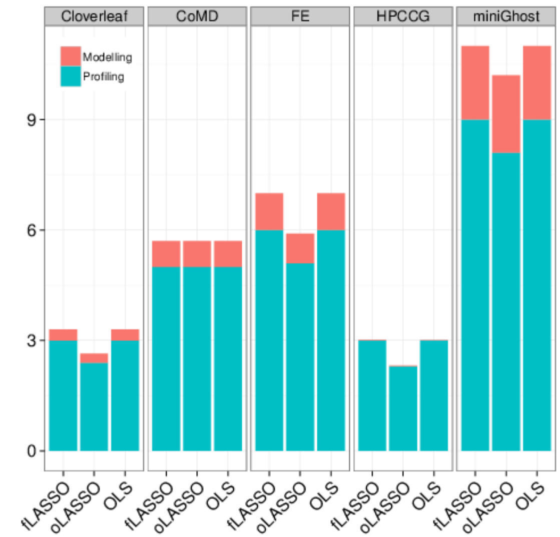
P. Garrigues and L. El Ghaoui. An homotopy algorithm for the Lasso with online observations. NIPS 2008

PEMOGEN – static analysis

- **Also integrated into LLVM compiler**
  - Automatic kernel detection and instrumentation (Loop Call Graph)
  - Static dataflow analysis reduces parameter space for each kernel



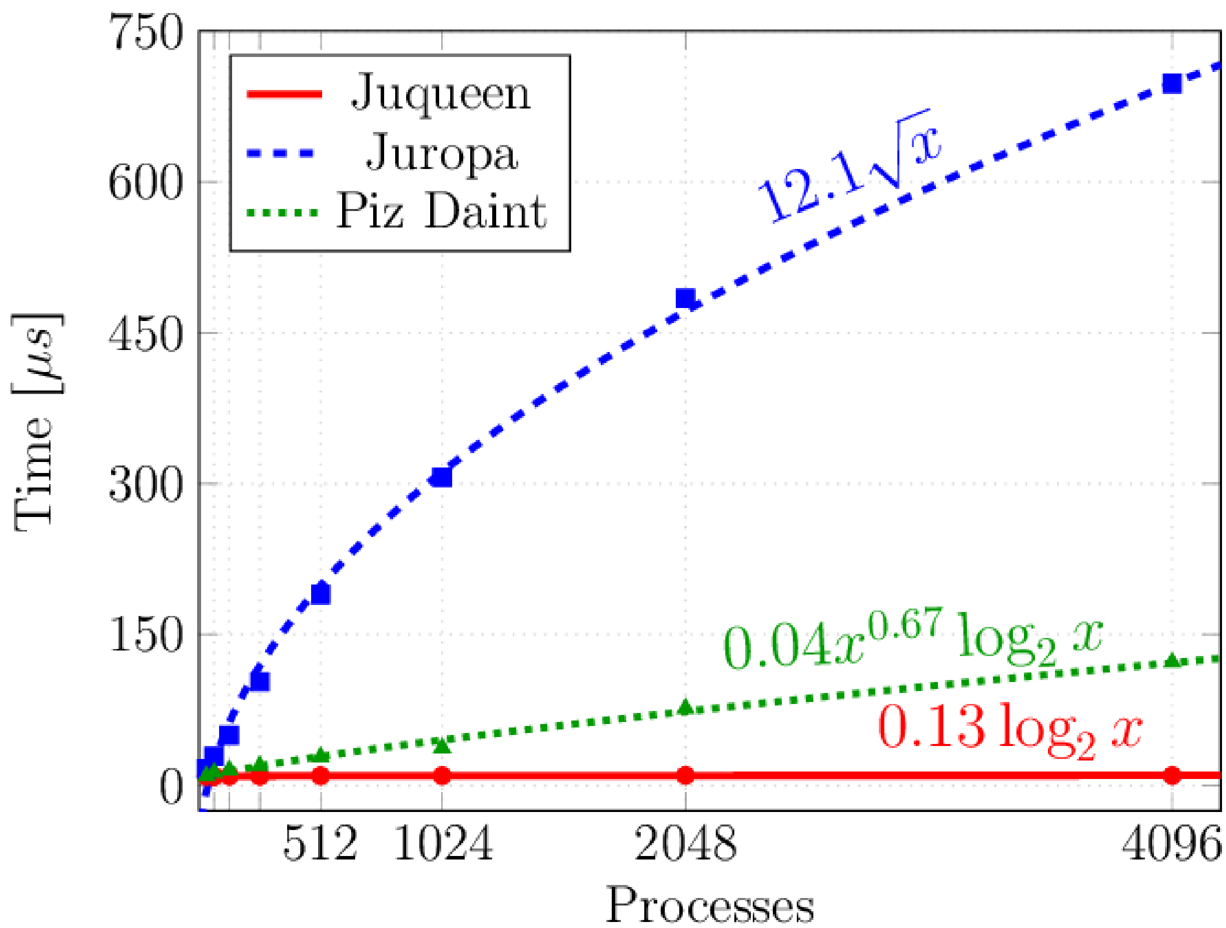
Quality: NAS UA and Mantevo MiniFE



Overhead: Mantevo

A. Bhattacharyya, T. Hoefler: PEMOGEN: Automatic Adaptive Performance Modeling during Program Runtime, PACT'14

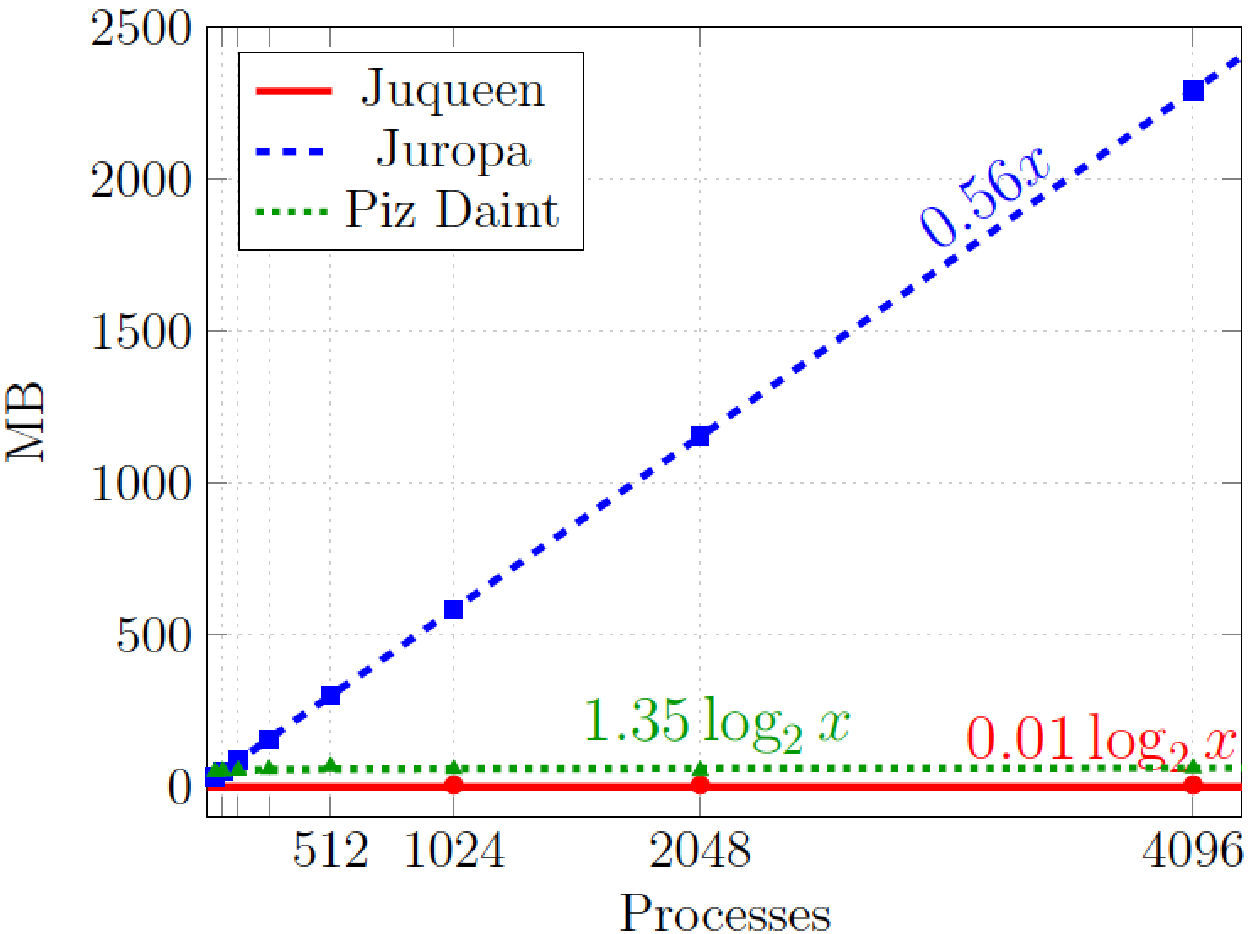
Use-case A: automatic testing (Allreduce time)



- **Divergence on Piz**
- **Daint is $O(p^{0.67})$, the highest of all three**

S. Shudler, A. Calotoiu, T. Hoefler, A. Strube, F. Wolf: Exascaling Your Library: Will Your Implementation Meet Your Expectations?, ICS'15

Use-case B: automatic testing (MPI memory size)



- **Linear memory consumption on Juropa**
- **ParaStation MPI**
- **uses RC over IB**

S. Shudler, A. Calotoiu, T. Hoefler, A. Strube, F. Wolf: Exascaling Your Library: Will Your Implementation Meet Your Expectations?, ICS'15

Performance Analysis 2.0 – Automatic Models

- **Is feasible**
  Still a long way to go …
- **Offers insight**
- **Requires low effort**
- **Improves code coverage**

SPPEXA

DFG

FNSNF

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

German Research School for Simulation Sciences

SPAA 2014

SC13 Denver, CO 2013

ICS

A. Calotoiu, T. Hoefler, M. Poke, F. Wolf: Using Automated Performance Modeling to Find Scalability Bugs in Complex Codes. *Supercomputing (SC13).*

T. Hoefler, G. Kwasniewski: Automatic Complexity Analysis of Explicitly Parallel Programs. *SPAA 2014.*

A. Bhattacharyya, T. Hoefler: PEMOGEN: Automatic Adaptive Performance Modeling during Program Runtime, *PACT 2014*

S. Shudler, A. Calotoiu, T. Hoefler, A. Strube, F. Wolf: Exascaling Your Library: Will Your Implementation Meet Your Expectations? *ICS 2015*

Backup

Counting Arbitrary Affline Loop Nests

- **Why affine loops?**
  - Closed form representation of the loop

```
x=x_0;                  // Initial assignment
while(c^T x < g)        // Loop guard
    x=Ax + b;           // Loop update
```

$$x(i, x_0) = L(i) \cdot x_0 + p(i)$$

$$n(x_0, c, g) = \arg\min_d (c^T \cdot x(d, x_0) \geq g)$$

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- **Example**

```
for ( k=j; k < m; k = k + j )
        veryComplicatedOperation(j,k);
```

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$while\ (\begin{pmatrix} 0 & 1 \end{pmatrix} x < m)\{$$

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\}$$

$$x(i, x_0) = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} x_0 + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$n(x_0) = \left\lceil \frac{m - k_0}{j_0} \right\rceil$$

where $x_0 = \begin{pmatrix} j_0 \\ k_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Loops

- **Multipath affine loops**

```
x=1;
while(x < n/p + 1) {
  y=x;
  while(y < m) { S1; y=2*y; }
  z=x;
  while(z < m) { S2; z= z + x; }
  x=2*x;
}
```